Now we have to worry about the cases 9/19

Draft 58 cont.

$E > 0$, $E = 0$, and $E < 0$.

For $E > 0$ the solution is $u = \frac{1}{R} \sinh[\Omega(\theta - \theta_0)]$

$$\Rightarrow \frac{du}{d\theta} = \frac{\Omega}{R} \cosh[\Omega(\theta - \theta_0)] \Rightarrow$$

$$\left(\frac{du}{d\theta}\right)^2 - \Omega^2 u = \frac{\Omega^2}{R^2}\left[\cosh^2 - \sinh^2\right] = \frac{\Omega^2}{R^2} > 0$$

Thus, in this case $r = 1/u \Rightarrow$

$$\boxed{r = \frac{R}{\sinh[\Omega(\theta - \theta_0)]}}$$

We see that $r = \infty$ at $\theta = \theta_0$, and $r \to 0$ as $\theta \to \infty$.

This gives a solution that in to the origin from $r = \infty$.

It can also be time reversed to give a solution that spirals out from the origin to infinity

For $E = 0$ the solution is $u = \frac{1}{R} \exp[\Omega(\theta - \theta_0)]$

$$\Rightarrow \frac{du}{d\theta} = \Omega u \Rightarrow \left(\frac{du}{d\theta}\right)^2 - \Omega^2 u = 0.$$

Thus, in this case $r = 1/u \Rightarrow$

$$r = R \exp[-\Omega(\theta - \theta_0)]$$

Solution spirals in to $r = 0$ as $\theta \to \infty$, and goes to $r = \infty$ as $\theta \to -\infty$.