Dragt 5? cont.

Integrating $\overset{\star\star}{\star\star}$ $\Rightarrow$ $\boxed{\dot\psi = -2(\omega/R)\rho + \alpha}$ $\overset{\star\star}{\star\star\star}$ 6/19

↑ a constant

$\Rightarrow$ $2R\omega\dot\psi = -4\omega^2\rho + 2R\omega\alpha$ and $\star\star\star$ becomes

$$m\ddot\rho = 4\omega^2\rho - 4\omega^2\rho + 2R\omega\alpha \Rightarrow$$

$$m\ddot\rho = 2R\omega\alpha \Rightarrow \boxed{\rho = \frac{R\omega\alpha}{m}t^2 + \beta t + \gamma} \quad \overset{\star\star\star}{\star\star\star}$$

Putting $\overset{\star\star}{\star\star\star}$ back in $\overset{\star\star}{\star\star\star}$ and integrating $\Rightarrow$

$$\boxed{\psi = \alpha t - \frac{2}{3}\frac{\omega^2\alpha}{m}t^3 - \frac{\omega}{R}\beta t^2 - \frac{2\omega}{R}\gamma t + \text{cnst}}$$

We see that ρ grows/decreases in time unless $\alpha = \underset{\dot\rho(0)}{\beta} = 0$, and therefore the circular solution is unstable.

b) Use the orbit equation

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2}\frac{d}{du}V\left(\frac{1}{u}\right).$$ Goldstein 3.34

$$V(r) = \frac{\lambda}{r^2} \Rightarrow V\left(\frac{1}{u}\right) = \lambda u^2$$ and

$$\frac{d}{du}V\left(\frac{1}{u}\right) = 2\lambda u \Rightarrow$$

$$\boxed{\frac{d^2u}{d\theta^2} + u\left[1 + \frac{2m\lambda}{L^2}\right] = 0} \quad \star$$