Dragt 5 ☆ cont

$\frac{\partial L}{\partial \dot\phi} = mr^2\dot\phi$, $\frac{\partial L}{\partial \phi} = 0 \Rightarrow$ $\boxed{r\ddot\phi + 2\dot r\dot\phi = 0}$ ☆☆

iii) Suppose $r = R$ + $\phi = \omega t$ is a solution:

Then ☆☆ is satisfied and ☆ $\Rightarrow 0 = \frac{2\lambda}{R^3} + R\omega^2$

$\Rightarrow \omega^2 = \frac{-2\lambda}{R^4} \Rightarrow \boxed{\omega = \sqrt{\frac{-2\lambda}{R^4}}}$

iv) Now write $r = R + \epsilon\rho$, $\phi = \omega t + \epsilon\psi \Rightarrow$

using ☆

$$\epsilon m\ddot\rho = \frac{2\lambda}{(R+\epsilon\rho)^3} + (R+\epsilon\rho)(\omega+\epsilon\dot\psi)^2 \Rightarrow$$

$$\epsilon m\ddot\rho = \frac{2\lambda}{R^3}\left(1+\frac{\epsilon\rho}{R}\right)^{-3} + (R+\epsilon\rho)(\omega^2 + 2\omega\epsilon\dot\psi) + O(\epsilon^2)$$

$$\Rightarrow \epsilon m\ddot\rho = \frac{2\lambda}{R^3} - \frac{6\lambda}{R^4}\epsilon\rho + R\omega^2 + \omega^2\epsilon\rho + R\,2\omega\epsilon\dot\psi + O(\epsilon^2)$$

cancel

$$\Rightarrow \epsilon m\ddot\rho = \left(-\frac{6\lambda}{R^4} - \frac{2\lambda}{R^4}\right)\epsilon\rho + 2R\omega\epsilon\dot\psi + O(\epsilon^2) \Rightarrow$$

☆☆☆ $\boxed{m\ddot\rho = 4\omega^2\rho + 2R\omega\dot\psi}$. And from ☆☆ we have

$$R\epsilon\ddot\psi + 2\epsilon\dot\rho\omega = 0 \Rightarrow$$

☆☆☆☆ $\boxed{\ddot\psi + 2(\omega/R)\dot\rho = 0}$