a circle as $t \to \pm\infty$. Let us solve for θ in this case. Now we have

$$\dot{\theta} = \pm \sqrt{\frac{-2\lambda}{m\left[R \pm (t-t_0)\sqrt{2E/m}\right]^4}} \Rightarrow$$

$$\theta(t) = \theta_0 \pm \int_{t_0}^{t} dt' \, \frac{\sqrt{\frac{-2\lambda}{m}}}{\left[R \pm (t'-t_0)\sqrt{\frac{2E}{m}}\right]^2} \Rightarrow$$

$$\theta(t) = \theta_0 \pm \sqrt{\frac{-2\lambda}{m}} \, \frac{1}{\mp\sqrt{\frac{2E}{m}}} \left[\frac{1}{R \pm (t'-t_0)\sqrt{\frac{2E}{m}}}\right]\Bigg|_{t_0}^{t}$$

$$\Rightarrow \boxed{\theta(t) = \theta_0 \pm \sqrt{\frac{-\lambda}{E}} \, (\mp) \left\{ \frac{1}{R \pm (t-t_0)\sqrt{\frac{2E}{m}}} - \frac{1}{R} \right\}}$$

signs correlated.

If we take the + sign here, then $\dot{r} > 0$ and we see $\displaystyle \theta(t) \underset{t\to\infty}{=} \theta_0 \pm \sqrt{\frac{-\lambda}{E}}\,\frac{1}{R}$, and the orbit is eventually a straight line with increasing r. If we take the − sign, $\dot{r} < 0$