

Dragt 58 cont.

2/19

$E = \frac{m}{2} (\dot{r})^2$ and a circular orbit

$r = R$ will be possible if $E = 0$.

So we get a circular orbit when

$$r = R \text{ and } L^2 = -2m\lambda \Rightarrow \left(m r^2 \dot{\theta} \right)^2 = -2m\lambda$$

$$\Rightarrow \boxed{r = R \text{ and } \dot{\theta} = \pm \sqrt{\frac{-2\lambda}{m R^4}}}$$
$$\Rightarrow \theta = \theta_0 \pm (t - t_0) \sqrt{\frac{-2\lambda}{m R^4}}$$

Now suppose we perturb this orbit slightly. If we slightly change $\dot{\theta}$ or r , we get $V_{\text{eff}} \neq 0$ and the orbit wanders far from its circular shape.

If we keep $L^2 = -2m\lambda$ and only change \dot{r} , we have

$$E = \frac{m}{2} (\dot{r})^2 \Rightarrow \dot{r} = \pm \sqrt{2E/m}$$

$$\Rightarrow \boxed{r(t) = R \pm (t - t_0) \sqrt{2E/m}}$$

Again the orbit does not stay near