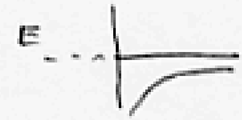
For the case $E = 0$ we get



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$$\frac{m}{2}(\dot{r}^2) + \frac{\sigma}{r^2} = 0 \Rightarrow (\dot{r})^2 = \left(-\frac{2\sigma}{m}\right)\frac{1}{r^2} \Rightarrow$$

$$\dot{r} = \sqrt{\frac{-2\sigma}{m}}\,\frac{1}{r} \Rightarrow \frac{dt}{dr} = \sqrt{\frac{m}{-2\sigma}}\, r \Rightarrow$$

$$t - t_0 = \frac{1}{2}\sqrt{\frac{m}{-2\sigma}}\left[r^2 - r_0^2\right] \Rightarrow r^2 - r_0^2 = \sqrt{\frac{-2\sigma}{m}}\, 2(t - t_0)$$

$$\Rightarrow \boxed{r = \left[r_0^2 + \sqrt{\frac{-8\sigma}{m}}\,(t - t_0)\right]^{1/2}} \qquad \dot{\theta} = \frac{L}{m r^2} \Rightarrow$$

$$\dot{\theta} = \frac{L}{m}\,\frac{1}{\left[r_0^2 + \sqrt{-8\sigma/m}\,(t - t_0)\right]} \Rightarrow$$

$$\tau = t' - t_0$$

$$\theta - \theta_0 = \frac{L}{m}\int_{t_0}^{t}\frac{dt'}{\left[r_0^2 + \sqrt{\frac{-8\sigma}{m}}\,(t' - t_0)\right]} \Rightarrow$$

$$\theta - \theta_0 = \frac{L}{m}\int_{0}^{t - t_0}\frac{d\tau}{\left[r_0^2 + \sqrt{\frac{-8\sigma}{m}}\,\tau\right]} \Rightarrow$$

$$\boxed{\theta - \theta_0 = \frac{L}{m}\sqrt{\frac{m}{-8\sigma}}\,\log\left(r_0^2 + \sqrt{\frac{-8\sigma}{m}}\,\tau\right)\Bigg|_0^{t - t_0}}$$