Dragt 58 cont.

$$r^2 - \frac{\sigma}{E} = r_0^2 - \frac{\sigma}{E} + 2\left[r_0^2 - \frac{\sigma}{E}\right]^{1/2}\sqrt{\frac{2E}{m}}\,(t-t_0) + \frac{2E}{m}(t-t_0)^2 \Rightarrow$$

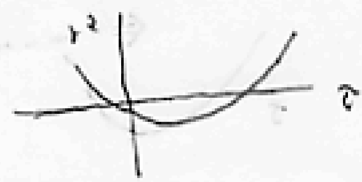
$$r^2 = r_0^2 + 2\left[r_0^2 - \frac{\sigma}{E}\right]^{1/2}\sqrt{\frac{2E}{m}}\,(t-t_0) + \frac{2E}{m}(t-t_0)^2 \Rightarrow$$

$$\boxed{r = \left\{ r_0^2 + 2\left[r_0^2 - \frac{\sigma}{E}\right]^{1/2}\sqrt{\frac{2E}{m}}\,(t-t_0) + \frac{2E}{m}(t-t_0)^2 \right\}^{1/2}}$$

Let us write r^2 in the form

$$r^2 = \underbrace{r_0^2}_{=a} + \underbrace{2\left[r_0^2 - \frac{\sigma}{E}\right]^{1/2}\sqrt{\frac{2E}{m}}}_{=b}\,\underbrace{(t-t_0)}_{=\tau} + \underbrace{\frac{2E}{m}}_{=c}\,\underbrace{(t-t_0)^2}_{\tau^2} \Rightarrow$$

$$r^2 = a + b\tau + c\tau^2 \quad \text{with } c > 0 \Rightarrow$$



A parabola that opens upward.

Find its minimum $\Rightarrow \dfrac{dr^2}{d\tau} = b + 2c\tau = 0 \Rightarrow \tau = -b/2c$

$$\Rightarrow (r_{min})^2 = a + b\left(\frac{-b}{2c}\right) + c\left(\frac{-b}{2c}\right)^2 = a - \frac{b^2}{4c} = \frac{1}{4c}\left[4ac - b^2\right]$$

But $4ac - b^2 = 4 r_0^2 \dfrac{2E}{m} - 4\dfrac{2E}{m}\left[r_0^2 - \sigma/E\right]$

$$= 4\,\frac{2E}{m}\left[r_0^2 - r_0^2 + \sigma/E\right] = \frac{8E}{m}(\sigma/E) < 0 \;!$$

So, we get $r = 0$ for finite τ.