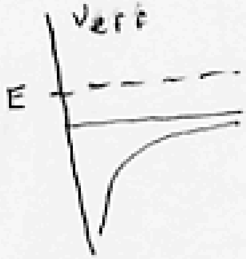
Now there are 3 subcases to consider:

$E < 0$, $E = 0$, and $E > 0$.

First do the case $E > 0 \Rightarrow$



$$\frac{m}{2}(\dot r)^2 + \frac{\sigma}{r^2} = E \Rightarrow$$

$$\dot r = \sqrt{\frac{2E}{m} - \frac{2\sigma}{m}\frac{1}{r^2}} = \sqrt{\frac{2E}{m}}\left[1 - \frac{\sigma}{E}\frac{1}{r^2}\right]^{1/2}$$

$$\Rightarrow dt = \sqrt{\frac{m}{2E}}\frac{dr}{\left[1 - \frac{\sigma}{E}\frac{1}{r^2}\right]^{1/2}} \Rightarrow$$

$$t - t_0 = \sqrt{\frac{m}{2E}}\int_{r_0}^{r}\frac{dr'}{\left[1 - \frac{\sigma}{E}\frac{1}{r'^2}\right]^{1/2}} \Rightarrow$$

$$t - t_0 = \sqrt{\frac{m}{2E}}\int_{r_0}^{r}\frac{r'\,dr'}{\left[(r')^2 - \frac{\sigma}{E}\right]^{1/2}} \Rightarrow$$

$$t - t_0 = \sqrt{\frac{m}{2E}}\left[(r')^2 - \frac{\sigma}{E}\right]^{1/2}\Bigg|_{r_0}^{r} \Rightarrow$$

$$t - t_0 = \sqrt{\frac{m}{2E}}\left\{\left[r^2 - \frac{\sigma}{E}\right]^{1/2} - \left[r_0^2 - \frac{\sigma}{E}\right]^{1/2}\right\} \Rightarrow$$

$$\left[r^2 - \frac{\sigma}{E}\right]^{1/2} = \left[r_0^2 - \frac{\sigma}{E}\right]^{1/2} + \sqrt{\frac{2E}{m}}(t - t_0) \Rightarrow$$