$$t^2 = \left(\frac{m}{2\sigma}\right) R^4 \left[\left(\frac{r}{R}\right)^2 - 1\right] \Rightarrow$$

$$\left(\frac{r}{R}\right)^2 - 1 = \frac{t^2}{\left(\frac{m}{2\sigma}\right) R^4} \Rightarrow \left(\frac{r}{R}\right)^2 = 1 + \frac{t^2}{\left(\frac{m}{2\sigma}\right) R^4}$$

$$\boxed{r(t) = R\sqrt{1 + \frac{2\sigma t^2}{mR^4}}}$$

To find θ use

$$L = mr^2\dot{\theta} \Rightarrow$$

$$\dot{\theta} = \frac{L}{mr^2} \Rightarrow \theta = \frac{L}{m}\int_0^t \frac{dt'}{R^2\left[1 + \frac{2\sigma t'^2}{mR^4}\right]} \Rightarrow$$

$$\theta = \frac{L}{mR^2}\frac{1}{\sqrt{\frac{2\sigma}{mR^4}}} \tan^{-1}\left[t'\sqrt{\frac{2\sigma}{mR^4}}\right]\Bigg|_0^t \Rightarrow$$

$$\boxed{\theta(t) = L\left(\frac{1}{2\sigma m}\right)^{1/2} \tan^{-1}\left[t\sqrt{\frac{2\sigma}{mR^4}}\right]}$$

The case $\sigma = 0$ has been done earlier.

What is left is the case $\sigma < 0 \Rightarrow$