

Dragt 58 cont.
So we can write

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$$\dot{r} = \left(\frac{2\sigma}{m}\right)^{1/2} \left[\frac{1}{R^2} - \frac{1}{r^2} \right]^{1/2}$$

Suppose we follow orbit from R to $\infty \Rightarrow \dot{r} > 0$

$$\text{and } dt = \left(\frac{m}{2\sigma}\right)^{1/2} \frac{dr}{\left[\frac{1}{R^2} - \frac{1}{r^2}\right]^{1/2}} \Rightarrow$$

$$t = \left(\frac{m}{2\sigma}\right)^{1/2} \int_R^r \frac{dr'}{\left[\frac{1}{R^2} - \frac{1}{(r')^2}\right]^{1/2}} \Rightarrow$$

$$t = \left(\frac{m}{2\sigma}\right)^{1/2} R \int_R^r \frac{r' dr'}{\left[r'^2 - R^2\right]^{1/2}}$$

$$\text{Let } r' = R\tau \Rightarrow dr' = R d\tau \Rightarrow$$

$$t = \left(\frac{m}{2\sigma}\right)^{1/2} R^2 \int_1^{r/R} \frac{\tau d\tau}{\left[\tau^2 - 1\right]^{1/2}} \Rightarrow$$

$$t = \left(\frac{m}{2\sigma}\right)^{1/2} R^2 \left[\tau^2 - 1 \right]^{1/2} \Big|_1^{r/R} \Rightarrow$$

$$t = \left(\frac{m}{2\sigma}\right)^{1/2} R^2 \left[\left(\frac{r}{R}\right)^2 - 1 \right]^{1/2} \Rightarrow$$