

For $E < 0$ the solution is $u = \frac{1}{R} \cosh k [\alpha(\theta - \theta_0)]$

$$\Rightarrow \frac{du}{d\theta} = -\frac{\alpha}{R} \sinh k [\alpha(\theta - \theta_0)] \Rightarrow$$

$$\left(\frac{du}{d\theta}\right)^2 - \alpha^2 u^2 = \frac{\alpha^2}{R^2} [\sinh^2 k - \cosh^2 k] = -\frac{\alpha^2}{R^2} < 0.$$

Thus, in this case $r = 1/u \Rightarrow$

$$r = \frac{R}{\cosh k [\alpha(\theta - \theta_0)]}$$

In this case the orbit spirals out of the origin as θ increases from $-\infty$, reaches $r = R$ when $\theta = \theta_0$, and spirals back in as $\theta \rightarrow +\infty$.