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a) $V = \lambda/r^2$. Can only have a circular orbit if the force is attractive $\Rightarrow \lambda < 0$.

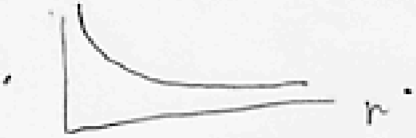
Following the Berkeley Mechanics Notes we write

$$E = \frac{m}{2}(\dot{r})^2 + \frac{L^2}{2mr^2} + \frac{\lambda}{r^2} = \frac{m}{2}(\dot{r})^2 + \frac{\left(\frac{L^2}{2m}+\lambda\right)}{r^2} \Rightarrow$$

$$E = \frac{m}{2}(\dot{r})^2 + V_{eff}(r) \quad \text{with} \quad V_{eff}(r) = \left(\frac{L^2}{2m}+\lambda\right)/r^2$$

If $\frac{L^2}{2m} + \lambda > 0$ [which gives $L^2 > -2m\lambda$],

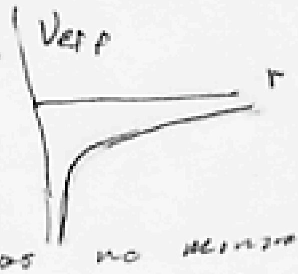
then $V_{eff}(r)$ has the graph



Clearly in this case $V_{eff}(r)$ has no minimum, and so there is no circular orbit.

If $\frac{L^2}{2m} + \lambda < 0$ [which gives $0 < L^2 < -2m\lambda$],

then $V_{eff}(r)$ has the graph



Again in this case $V_{eff}(r)$ has no minimum and so there is no circular orbit.

Finally, suppose $L^2 = -2m\lambda$ [which gives $V_{eff} = 0$]. Then we have