Drag7 5> cont

We now want to solve this for $\beta(\alpha)$. Since

$$\frac{d\alpha}{d\beta} = 1 - \varepsilon \cos\beta$$

is never zero for $0 < \varepsilon < 1$, this expression is invertible. So we expand β in a Fourier series

$$\beta - \alpha = \sum_{n=1}^{\infty} u_n \sin n\alpha$$

and u_n is of course given by $u_n = \frac{2}{\pi}\int_0^{\pi}[\beta(\alpha) - \alpha]\sin n\alpha \, d\alpha$

$$= -\frac{2}{n\pi}\int_0^{\pi}[\beta(\alpha) - \alpha]\, d\cos n\alpha \, .$$

Integrate by parts to get

$$-\frac{2}{n\pi}[\beta(\alpha) - \alpha]\cos n\alpha\Big|_0^{\pi} + \frac{2}{n\pi}\int_0^{\pi}\cos n\alpha \, d[\beta(\alpha) - \alpha]$$

When $\alpha = \pi$ we have $\pi = \beta - \varepsilon \sin\beta$. $\beta = \pi$ is a solution to this and because $d\alpha/d\beta > 0$ it is the only solution. So $\beta(\pi) - \pi = 0$. Similarly for $\alpha = 0$. So the first term $(\;)\big|_0^{\pi}$ is zero and we get

$$u_n = \frac{2}{n\pi}\int_0^{\pi}\cos n\alpha \, d\beta(\alpha) - \underbrace{\frac{2}{n\pi}\int_0^{\pi}\cos n\alpha \, d\alpha}_{0 \text{ since } n \geq 1}$$