Dragt 57 cont.

Putting these together gives $r = r_0 + \frac{\epsilon^2 r_0}{1-\epsilon^2} - a\epsilon\cos\beta$

or $r = \frac{r_0}{1-\epsilon^2} - a\epsilon\cos\beta$ or $\boxed{r - a = -a\epsilon\cos\beta}$

Therefore, $dr = a\epsilon\sin\beta\, d\beta$ and also

$$\left[a^2\epsilon^2 - (r-a)^2\right]^{\frac{1}{2}} = \left[a^2\epsilon^2 - a^2\epsilon^2\cos^2\beta\right]^{\frac{1}{2}} = a\epsilon\sin\beta!$$

So the integral for t becomes the simple result

$$t = \frac{T_0}{2\pi}\frac{1}{a}\int_0^\beta \frac{\overset{r'}{\overset{\shortparallel}{(a - a\epsilon\cos\beta')}}\; a\epsilon\sin\beta'\, d\beta'}{a\epsilon\sin\beta'}$$ which gives

$$t = \frac{T_0}{2\pi}\int_0^\beta (1 - \epsilon\cos\beta')\, d\beta' = \frac{T_0}{2\pi}\left[\beta - \epsilon\sin\beta\right]$$

Thus $\boxed{\alpha = \frac{2\pi t}{T_0} = \beta - \epsilon\sin\beta}$ QED