Dragt 57 cont.

$$\dot{r} = \frac{2\pi}{T_0} \frac{a}{r} \left[-r^2 - a^2(1-\epsilon^2) + 2ar \right]^{\frac{1}{2}} \quad \text{or}$$

$$\boxed{\dot{r} = \frac{2\pi}{T_0} \frac{a}{r} \left[a^2\epsilon^2 - (r-a)^2 \right]^{\frac{1}{2}}}$$

Noting that $\dot{r} = \frac{dr}{dt}$, we now get

$$dt = \frac{T_0}{2\pi} \frac{1}{a} \frac{r\,dr}{\left[a^2\epsilon^2 - (r-a)^2 \right]^{\frac{1}{2}}} \quad \text{which gives}$$

$$\boxed{t = \frac{T_0}{2\pi} \frac{1}{a} \int_{r_{min}}^{r} \frac{r'\,dr'}{\left[a^2\epsilon^2 - (r'-a)^2 \right]^{1/2}}}$$

Here t is measured from the moment of pericenter, and

$$r_{min} = r\Big|_{pericenter} = \frac{r_0}{1+\epsilon}$$

How do we do this integral?

We found earlier that $r + \epsilon r\cos\theta = r_0$ or $r = r_0 - \epsilon r\cos\theta$

and $r\cos\theta + \frac{\epsilon r_0}{(1-\epsilon^2)} = a\cos\beta$ or $\epsilon r\cos\theta = a\epsilon\cos\beta - \frac{\epsilon^2 r_0}{1-\epsilon^2}$.