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Now we are ready to derive Kepler's equations: From Class lectures or hand out notes we have

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + V(r) \quad \text{with} \quad V(r) = \frac{-g}{r}. \quad \text{Solving for } \dot{r} \Rightarrow$$

$$\dot{r} = \left(\frac{2}{m}\right)^{\frac{1}{2}} \left[E - \frac{L^2}{2mr^2} + \frac{g}{r} \right]^{\frac{1}{2}}.$$

We also have $E = -g/2a$

$L^2 = m g r_0$ from notes.

So we can also write

$$\dot{r} = \left(\frac{2}{m}\right)^{\frac{1}{2}} \left[-\frac{g}{2a} - \frac{m g r_0}{2mr^2} + \frac{g}{r} \right]^{\frac{1}{2}} \quad \text{or}$$

$$\dot{r} = \left(\frac{g}{m}\right)^{\frac{1}{2}} \frac{1}{a^{\frac{1}{2}} r} \left[-r^2 - a r_0 + 2ar \right]^{\frac{1}{2}}.$$

Next, $T_0 = \dfrac{2\pi a^{\frac{3}{2}}}{(g/m)^{\frac{1}{2}}}$ from notes.

$$\Downarrow \left(\frac{g}{m}\right)^{\frac{1}{2}} = \frac{2\pi a^{\frac{3}{2}}}{T_0}$$

So we also have

$$\dot{r} = \frac{2\pi}{T_0} \frac{a}{r} \left[-r^2 - a r_0 + 2ar \right]^{\frac{1}{2}}.$$

Also, $a r_0 = a^2(1 - \epsilon^2)$ from notes.

Putting in for $a r_0 \Rightarrow$