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The equation for a bound orbit reads $\boxed{r = \frac{r_0}{1+\epsilon\cos\theta}}$

or $\boxed{r + \epsilon r\cos\theta = r_0}$. Introduce cartesian coordinates: $x = r\cos\theta$, $y = r\sin\theta$

$\Rightarrow \sqrt{x^2+y^2} + \epsilon x = r_0$. Now square out: $\Rightarrow x^2+y^2 = (r_0 - \epsilon x)^2 = r_0^2 - 2r_0\epsilon x + \epsilon^2 x^2$.

Collecting terms and normalizing, we get

$$\boxed{\left[\frac{(1-\epsilon^2)}{r_0}\right]^2 \left[x + \frac{\epsilon r_0}{(1-\epsilon^2)}\right]^2 + \left[\frac{(1-\epsilon^2)^{\frac{1}{2}}}{r_0}\right]^2 y^2 = 1}$$

But we have $a = \frac{r_0}{1-\epsilon^2}$, $b = \frac{r_0}{(1-\epsilon^2)^{1/2}}$. Let $\xi = x + \frac{\epsilon r_0}{(1-\epsilon^2)}$

Then we get

$\boxed{\frac{\xi^2}{a^2} + \frac{y^2}{b^2} = 1}$ which is the standard equation for an ellipse.

Now define an angle β by writing $\boxed{\frac{\xi}{a} = \cos\beta\,,\ \frac{y}{b} = \sin\beta}$