Next examine non circular orbits to see if $\exists$ restrictions on a, b, + ν.

Then $\ddot{u} + u = -m^2 L^{-2} G'(u) \Rightarrow \ddot{u} + u = -m^2 L^{-2} \nu a u^{\nu-1}$. Write

$u = u_0(1+\epsilon) \Rightarrow u_0 [\ddot{\epsilon} + 1 + \epsilon] = -m^2 L^{-2} \nu a u_0^{\nu-1} (1+\epsilon)^{\nu-1}$.

Define u_0 so that $\boxed{u_0 = -m^2 L^{-2} \nu a u_0^{\nu-1}} \Rightarrow \boxed{\ddot{\epsilon} + \epsilon = (1+\epsilon)^{\nu-1} - 1}$

Expand $(1+\epsilon)^{\nu-1} = 1 + (\nu-1)\epsilon + \sum_2^\infty \lambda_n \epsilon^n \Rightarrow$

$$\text{I} \quad \boxed{\ddot{\epsilon} + (2-\nu)\epsilon = \sum_2^\infty \lambda_n \epsilon^n}$$

Now, for nearly circular orbits, $\epsilon \approx 0$ + we get

$$\ddot{\epsilon} + (2-\nu)\epsilon = 0 + O(\epsilon^2)$$

We see that the "precession frequency" is independent of u_0 for small oscillations as expected. We now require that the oscillation frequency of the nonlinear equation I be <u>independent</u> of amplitude, i.e. we require that $\omega^2 = \omega_0^2 = 2-\nu$ even when $\epsilon \neq 0$! An esoteric result from nonlinear oscillation theory states that a <u>necessary</u> condition is that

$$\boxed{\frac{5}{6}\lambda_2^2/\omega_0^2 + \frac{3}{4}\lambda_3 = 0}$$

Putting in $\lambda_2 = \frac{(\nu-1)(\nu-2)}{2!}$

$\lambda_3 = \frac{(\nu-1)(\nu-2)(\nu-3)}{3!}$

$$\Rightarrow \frac{5}{6}\frac{(\nu-1)^2(\nu-2)^2}{4(2-\nu)} + \frac{3}{4}\frac{(\nu-1)(\nu-2)(\nu-3)}{6} = 0 \Rightarrow (\nu-2)^2(\nu-1)[3(\nu-3) - 5(\nu-1)] = 0 \Rightarrow \boxed{\nu = 1, 2, + -2}$$

$\therefore V \sim \frac{1}{r}, \frac{1}{r^2}, + r^2$. $\frac{1}{r^2}$ is ruled out by stability $\Rightarrow$ $V(r) = -\frac{a}{r}$ and ar^2, $a>0$ remain and actually work as you saw in the movie

Note $\frac{1}{r^2}$ gives spirals, + $\therefore$ it is ruled out