which gives $G[a\cos(\phi+b)] = -\frac{L^2a^2}{2m} + E$ which only makes sense if G is constant, + we again have no force!

The 3'd possibility is the most interesting: We have

$$\frac{kH\,G''(kH)}{G'(kH)} = \frac{H\,G''(H)}{G'(H)} \quad \text{or} \quad \frac{x\,G''(x)}{G'(x)} = \frac{H\,G''(H)}{G'(H)}$$

if we put $kH = x$. Since this is true for all x, we must have

$$\boxed{\frac{x\,G''(x)}{G'(x)} = \text{cnst} = \nu - 1}$$

This differential equation has the solution $\frac{G''(x)}{G'(x)} = \frac{\nu-1}{x} \Rightarrow \log G'(x) = (\nu-1)\log x + \text{cnst}$

$$\Rightarrow G'(x) = \text{cnst}\, x^{\nu-1} \Rightarrow \boxed{G(x) = a x^{\nu} + b}$$ $a + b$ arbitrary.

Thus, $$\boxed{V(r) = a r^{-\nu} + b}$$ $a, \nu, + b$ arbitrary