

IF $G'(u) \equiv 0$, then $G(u) = \text{const} = V(r)$ and there is
no force at all! Differentiate again to get

$$(\ddot{u} + \dot{u}) / G'(u) - (\ddot{u} + u) G''(u) \dot{u} / (G'(u))^2 = 0 \quad \text{or}$$

$$(\ddot{u} + \dot{u}) G'(u) = (\ddot{u} + u) G''(u) \dot{u}$$

This is an equation for the orbit which does not involve any initial conditions

Now put $u = kH$. Since this is an orbit, we must have

$$k(\ddot{H} + \dot{H}) G'(kH) = k^2(\ddot{H} + H) \dot{H} G''(kH) \quad \text{or}$$

$$(\ddot{H} + \dot{H}) G = (\ddot{H} + H) \dot{H} k G''(kH) / G'(kH)$$

which must be true for all k .

Putting $k=1$ we get

$$(\ddot{H} + \dot{H}) = (\ddot{H} + H) \dot{H} G''(H) / G'(H) \quad \text{Comparing we find}$$

$$0 = (\ddot{H} + H) \dot{H} \left[\frac{k G''(kH)}{G'(kH)} - \frac{G''(H)}{G'(H)} \right]$$