51.

(b) Spse $v(t=0) = v_0$. Then at the beginning we would have

$$\delta \int \sqrt{\frac{1+y'^2}{y+d^2}}\, dx = 0 \qquad d^2 = \text{const} = v_0^2/2g$$

which would lead in the same way to

$$\frac{y'^2}{\sqrt{(y+d^2)(1+y'^2)}} - \sqrt{\frac{1+y'^2}{y+d^2}} = c_1$$

$$\Rightarrow \quad (y+d^2)(1+y'^2) = C_2 > 0.$$

Now if we define $\bar{y} = y + d^2$

all the same results follow. Then we get

$$x = \alpha(\phi - \sin\varphi)$$

$$y = \alpha(1-\cos\varphi) - d^2$$

$$= \alpha(1-\cos\varphi) - \frac{v_0^2}{2g}.$$