51.

Let $u = \sqrt{y}$ $\quad du = \frac{1}{2\sqrt{y}}\, dy$

$$x = 2\int \frac{u^2\,du}{\sqrt{a^2-u^2}} \qquad \underline{a^2 = c_2 > 0}$$

$$x + b = 2\left[-\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)\right] \qquad \underline{\text{By CRC}}$$

$\nwarrow$ a is the $\underline{\underline{\text{pos}}}$ $\sqrt{\ }$ of c_2.

$$-\sqrt{y}\sqrt{a^2-y^2} + a^2\sin^{-1}\left(\frac{\sqrt{y}}{a}\right) = x + b$$

$$y(x=0) = 0 \quad \Rightarrow \quad b = 0$$

Now we know that y goes between $y=0$ and $y = c_2$ as noted at the bottom of the last page. So we can parameterize y by

$$y = \frac{a^2}{2}(1-\cos\varphi). \qquad 0 \le \varphi \le \frac{\pi}{2}.$$

$$\rightarrow 0 \le y \le a^2 = c_2$$

Thus $$x = -\sqrt{2\alpha^2 - 2\alpha^2\cos^2\varphi - \alpha^2(1-2\cos\varphi+\cos^2\varphi)} + \alpha\cos^{-1}\left(\frac{\alpha - \alpha(1-\cos\varphi)}{\alpha}\right)$$

where $\alpha = a^2/2$

$$x = -\alpha\sqrt{1-\cos^2\varphi} + \alpha\cos^{-1}(\cos\varphi)$$

$$\boxed{\begin{aligned} x &= \alpha(\phi - \sin\varphi) \\ y &= \alpha(1-\cos\varphi)\end{aligned}}$$

since $0 \le \varphi \le \frac{\pi}{2}$

$\underline{\text{Cycloid!}}$