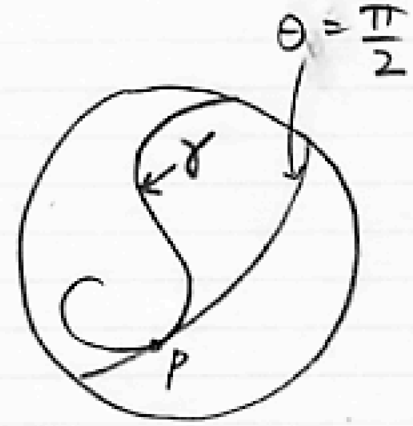
$$\theta'^2 - \theta'^2 - \sin^2\theta = -\frac{1}{K}\sqrt{\theta'^2 + \sin^2\theta} \quad \text{or}$$

$$K\sin^2\theta = \sqrt{\theta'^2 + \sin^2\theta}$$

Now it should be clear that we can rotate coordinates (θ, φ) on the sphere such that at a given point P on a curve γ, γ is tangent to the curve $\{\theta = \frac{\pi}{2}\}$. I.e. at the point P, $\theta = \frac{\pi}{2}$ and $\theta'(\varphi) = 0$.



This fixes the constant K to have value 1, so

$$\sin^4\theta = \theta'^2 + \sin^2\theta$$

$$\theta'^2 = \sin^4\theta - \sin^2\theta = -\sin^2\theta\cos^2\theta$$

$$\theta'^2 = -\frac{1}{4}\sin^2(2\theta)$$

But LHS ≥ 0, RHS ≤ 0 so both must be 0.

$$\theta' = 0 \quad \& \quad 2\theta = \pi$$

so $\boxed{\theta = \frac{\pi}{2} = \text{const wrt } \varphi}$

which is clearly a great circle.

See the next few sheets for a previous graders' quite different solution to the same problem.