Problem 48 cont:

c) Suppose $n = 3$. Then we have

$$\sum_{b=1}^{3} C_b(z)\, dz_b = \vec{F}(\vec{r}) \cdot d\vec{r} \quad \text{and}$$

$$(\vec{\nabla} \times \vec{F})_1 = \frac{\partial F_2}{\partial x_3} - \frac{\partial F_3}{\partial x_2}, \text{ etc.}$$

So, we have that there exists a ϕ such that $\vec{F} = \nabla \phi$ if and only if $\vec{\nabla} \times \vec{F} = 0$.