47 cont

Also note that $[n_0^2 - 2K]^{-1/2} = -\frac{1}{\mathcal{H}}$

and $\boxed{\mathcal{H} = \text{const}}$ since $\frac{\partial \mathcal{H}}{\partial z} = 0$.

Let $\zeta = [n_0^2 - 2K]^{-1/2} z$ be a new independent variable. Then we get

$$\dot{x} \stackrel{\text{def}}{=} \frac{dx}{d\zeta} = \frac{dx}{dz}\frac{dz}{d\zeta} = [n_0^2 - 2K]^{-1/2} [n_0^2 - 2K]^{+1/2} \frac{\partial K}{\partial p_x}$$

$$\Rightarrow \boxed{\dot{x} = \frac{\partial K}{\partial p_x}, \quad \dot{p}_x = -\frac{\partial K}{\partial x} \quad \text{etc.}}$$

But $K = \frac{p_x^2 + p_y^2}{2} + \frac{\omega^2}{2}(x^2 + y^2)$, A simple Harmonic Oscillator

where $\omega = n_0 \alpha \Rightarrow$

$$\dot{x} = p_x, \quad \dot{p}_x = -\omega^2 x \quad \Rightarrow \ddot{x} + \omega^2 x = 0$$

$$\Rightarrow \boxed{\begin{aligned} x &= A_x \sin(\omega\zeta + \phi_x) \\ y &= A_y \sin(\omega\zeta + \phi_y) \end{aligned}}$$