47. a) Done in class and in Draft notes.

b) $L = n(x,y,z)\left[1 + (x')^2 + (y')^2\right]^{1/2}$

$$p_x = \frac{\partial L}{\partial x'} = \frac{n(\vec{r})\,x'}{\left[1 + (x')^2 + (y')^2\right]^{1/2}}$$

$$p_y = \frac{\partial L}{\partial y'} = \frac{n(\vec{r})\,y'}{\left[1 + (x')^2 + (y')^2\right]^{1/2}}$$

c) $H = p_x x' + p_y y' - L =$

$$\frac{n(\vec{r})\left[(x')^2 + (y')^2\right]}{\left[1 + (x')^2 + (y')^2\right]^{1/2}} - n\left[1 + (x')^2 + (y')^2\right]^{1/2}$$

$$= \frac{-n}{\left[1 + (x')^2 + (y')^2\right]^{1/2}}.$$

But, $p_x^2 + p_y^2 = \dfrac{n^2\left[(x')^2 + (y')^2\right]}{\left[1 + (x')^2 + (y')^2\right]} \Rightarrow$

$$n^2 - p_x^2 - p_y^2 = n^2\left[1 - \frac{(x')^2 + (y')^2}{1 + (x')^2 + (y')^2}\right] \Rightarrow$$