

46. Let $y = y(x)$ describe the path of the light beam. Then, according to Fermat, the beam follows a path governed by the Lagrangian

$$\mathcal{L} = n(x, y) \sqrt{1 + (y')^2}$$

Making the approximation

$$n = n(h) + n'(h)(y-h) \text{ and}$$

$$[1 + (y')^2]^{1/2} = 1 + \frac{1}{2}(y')^2 \Rightarrow$$

$$\mathcal{L} = [n(h) + n'(h)(y-h)] [1 + \frac{1}{2}(y')^2] \Rightarrow$$

$$\mathcal{L} = \text{const} + n'(h)y + n(h)(\frac{1}{2})(y')^2 + \text{HOT.}$$

$$\frac{d}{dx} \frac{\partial \mathcal{L}}{\partial y'} - \frac{\partial \mathcal{L}}{\partial y} \Rightarrow n(h)y'' - n'(h) = 0$$

$$\therefore y(x) = a + bx + \frac{1}{2} \frac{n'(h)}{n(h)} x^2 \Rightarrow$$

Putting midpoint at $x=0$

$$y(x) = \left[h - \frac{1}{8} \frac{n'(h)}{n(h)} d^2 \right] + \frac{1}{2} \frac{n'(h)}{n(h)} x^2$$

Note $y(\pm d/2) = h$.