$$\mathcal{H} = p_\rho \dot{\rho} + p_\phi \dot{\phi} + p_z \dot{z} - \mathcal{L}$$

$$= \frac{1}{2} m \left[\dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2 \right] \Rightarrow$$

$$\boxed{\mathcal{H} = \frac{p_\rho^2}{2m} + \frac{p_\phi^2}{2m\rho^2} + \frac{\left[p_z + q \frac{\mu_0 I}{2\pi} \log \rho \right]^2}{2m}}$$

d) Observe that $\frac{\partial \mathcal{H}}{\partial \phi} = 0 \Rightarrow p_\phi = \text{const}$

$$\frac{\partial \mathcal{H}}{\partial z} = 0 \Rightarrow p_z = \text{const}$$

$$\frac{\partial \mathcal{H}}{\partial t} = 0 \Rightarrow \mathcal{H} = \text{const}$$

Thus, we have the result

$$\frac{1}{2} m \dot{\rho}^2 + \frac{p_\phi^2}{2m\rho^2} + \frac{\left[p_z + q \frac{\mu_0 I}{2\pi} \log \rho \right]^2}{2m} = \text{const}$$

This looks like 1-dimensional motion in the potential $V_{\text{effective}}(\rho) = \frac{p_\phi^2}{2m\rho^2} + \frac{\left[p_z + \frac{q\mu_0 I}{2\pi} \log \rho \right]^2}{2m}$

So, we can use "Energy Conservation" to get $\rho(t)$.