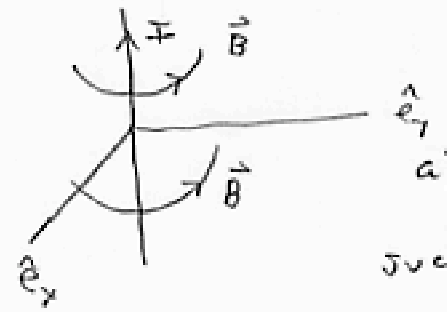
45. For an $\infty$ long straight wire we expect a magnetic field as shown below:

$$\vec{B} = \hat{e}_\phi \frac{\mu_0 I}{2\pi} \frac{1}{\rho}$$




a) We need to find an $\vec{A}$ such that $\vec{B} = \vec{\nabla} \times \vec{A}$.

Make the Ansatz $\vec{A} = A_z \hat{e}_z \Rightarrow$

$$B_\phi = -\frac{\partial A_z}{\partial \rho} \Rightarrow \boxed{\vec{A} = -\hat{e}_z \frac{\mu_0 I}{2\pi} \log \rho}$$

where $\rho = \sqrt{x^2 + y^2}$.

b) In cylindrical coordinates we have

$$T = \frac{1}{2} m \left[ \dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2 \right] \quad \text{and}$$

$$\mathcal{L} = T + q \vec{v} \cdot \vec{A} - q \psi \Rightarrow$$

$$\boxed{\mathcal{L} = \frac{1}{2} m \left[ \dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2 \right] - q \dot{z} \frac{\mu_0 I}{2\pi} \log \rho}$$

c) To find H, we compute:

$$p_\rho = \frac{\partial \mathcal{L}}{\partial \dot{\rho}} = m \dot{\rho}, \quad p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m \rho^2 \dot{\phi}$$

$$p_z = \frac{\partial \mathcal{L}}{\partial \dot{z}} = m \dot{z} - q \frac{\mu_0 I}{2\pi} \log \rho$$