

Now, assuming $r \neq s$, integrate to get 49 cont. 3/4

$$\frac{1}{1-\frac{r}{s}} \left[(p^F)^{1-\frac{r}{s}} - (p^A)^{1-\frac{r}{s}} \right] = -\lambda r \left(\frac{H}{\lambda} \right)^{\frac{r}{s}} (t^s - t^r) \Rightarrow$$

$$\left[(p^F)^{\frac{r-s}{r}} - (p^A)^{\frac{r-s}{r}} \right] = \lambda (-r) \left(1 - \frac{r}{s}\right) \left(\frac{H}{\lambda} \right)^{\frac{r}{s}} (t^s - t^r) \Rightarrow$$

$$\left[(p^F)^{\frac{r-s}{r}} - (p^A)^{\frac{r-s}{r}} \right] = \lambda (s-r) (t^s - t^r) \left[(q^A)^r (p^A)^s \right]^{\frac{r-1}{r}} \Rightarrow$$

$$(p^F)^{\frac{r-s}{r}} = (p^A)^{\frac{r-s}{r}} \left[1 + \lambda (s-r) (t^s - t^r) (q^A)^{r-1} (p^A)^{\frac{s-r}{r} + \frac{rs-r}{r}} \right]$$

$$\Rightarrow (p^F)^{\frac{r-s}{r}} = (p^A)^{\frac{r-s}{r}} \left[1 + \lambda (s-r) (t^s - t^r) (q^A)^{r-1} (p^A)^{s-1} \right] \Rightarrow$$

$$\boxed{p^F = p^A \left[1 + \lambda (s-r) (t^s - t^r) (q^A)^{r-1} (p^A)^{s-1} \right]^{\frac{r}{r-s}}} \quad (1.4.23)$$

Note: Let us compute $\lambda (q^F)^r (p^F)^s$ using (1.4.22) + (1.4.23).

We get $\lambda (q^F)^r (p^F)^s = \lambda (q^A)^r (p^A)^s \otimes$

$$\left[1 + \lambda (s-r) (t^s - t^r) (q^A)^{r-1} (p^A)^{s-1} \right]^{\left(\frac{s}{s-r}\right)(r) + \left(\frac{r}{r-s}\right)s}$$

But, $\frac{rs}{s-r} + \frac{rs}{r-s} = 0 \Rightarrow \boxed{\lambda (q^F)^r (p^F)^s = \lambda (q^A)^r (p^A)^s}$,

Which is what we expect since H is conserved!