

$$44. [D1.4.2] \quad H = \lambda g^r p^s \quad (1.4.15) \Rightarrow$$

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$$\dot{q} = \frac{\partial H}{\partial p} = \lambda s g^r p^{s-1}, \quad \dot{p} = -\frac{\partial H}{\partial q} = -\lambda r g^{r-1} p^s.$$

(1.4.16)

(1.4.17)

Equations (1.4.18) and (1.4.19) are derived in the text.

Rewrite  $\dot{q} = \lambda s \left(\frac{H}{\lambda}\right)^{\frac{s-1}{s}} g^{r/s}$  (1.4.29) as

$$\frac{dq}{dt} = \lambda s \left(\frac{H}{\lambda}\right)^{\frac{s-1}{s}} g^{r/s} \Rightarrow \frac{dq}{g^{r/s}} = \lambda s \left(\frac{H}{\lambda}\right)^{\frac{s-1}{s}} dt \Rightarrow$$

$$\int_{q^i}^{q^f} \frac{dq}{g^{r/s}} = \lambda s \left(\frac{H}{\lambda}\right)^{\frac{s-1}{s}} (t^f - t^i). \quad *$$

$$\text{If } r \neq s, \quad \int_{q^i}^{q^f} \frac{dq}{g^{r/s}} = \int_{q^i}^{q^f} dq g^{-r/s}$$

$$= \frac{1}{-r/s + 1} g^{-r/s + 1} \Big|_{q^i}^{q^f} = \frac{1}{1 - r/s} \left[ (q^f)^{1 - r/s} - (q^i)^{1 - r/s} \right]$$

$$= \frac{1}{1 - r/s} \left[ (q^f)^{\frac{s-r}{s}} - (q^i)^{\frac{s-r}{s}} \right]. \quad \text{Thus, we have}$$

$$\left[ (q^f)^{\frac{s-r}{s}} - (q^i)^{\frac{s-r}{s}} \right] = (1 - \frac{r}{s}) \lambda s \left(\frac{H}{\lambda}\right)^{\frac{s-1}{s}} (t^f - t^i) \Rightarrow$$