43. $f = e^x \sin y$

a) $u = \partial f/\partial x = e^x \sin y$

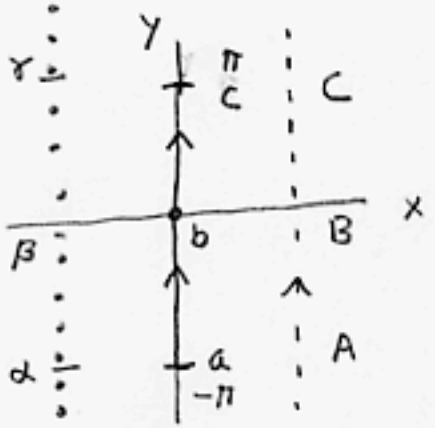
$v = \partial f/\partial y = e^x \cos y$

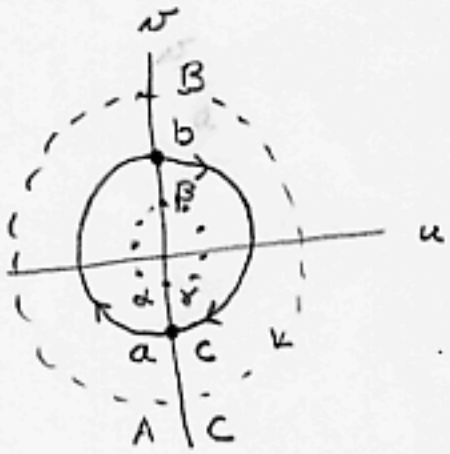
$$du = e^x \sin y\, dx + e^x \cos y\, dy$$
$$dv = e^x \cos y\, dx - e^x \sin y\, dy \quad \Rightarrow$$

$$\begin{pmatrix} du \\ dv \end{pmatrix} = \begin{pmatrix} e^x \sin y & e^x \cos y \\ e^x \cos y & -e^x \sin y \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$\det \begin{pmatrix} e^x \sin y & e^x \cos y \\ e^x \cos y & -e^x \sin y \end{pmatrix} = -e^{2x} \neq 0.$$

b)





$u = e^x \sin y \quad v = e^x \cos y$

$\Rightarrow u^2 + v^2 = e^{2x}$

Evidently lines of constant x are mapped into circles with x, y and x, $y \pm 2n\pi$ mapping to the same u, v.