Draft 42 cont. Again expand about $\theta = 0 \Rightarrow$

$$L = \frac{1}{2} M a^2 \dot{\theta}^2 - \frac{k a l_0 \sqrt{2}}{8} \theta^2 + M \Omega \Lambda \cos \Omega t \, a \dot{\theta} + \text{terms of order } 3$$

Now work out the eqns of motion:

$$\frac{\partial L}{\partial \dot{\theta}} = M a^2 \dot{\theta} + M a \Omega \Lambda \cos \Omega t \,, \quad \frac{\partial L}{\partial \theta} = -\frac{k a l_0 \sqrt{2}\, \theta}{4}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = M a^2 \ddot{\theta} - M a \Omega^2 \Lambda \sin \Omega t \,. \quad \therefore \text{ eqns of motion are}$$

$$M a^2 \ddot{\theta} - M a \Omega^2 \Lambda \sin \Omega t + \frac{k a l_0 \sqrt{2}\, \theta}{4} = 0 \quad \text{or}$$

$$\ddot{\theta} + \frac{k a l_0 \sqrt{2}}{4 M a^2} \theta = \Omega^2 \frac{\Lambda}{a} \sin \Omega t \,, \quad \text{or}$$

$$\boxed{\ddot{\theta} + \omega^2 \theta = \frac{\Omega^2 \Lambda}{a} \sin \Omega t}$$