c) Suppose the support point moves up and down.

Let $z_c = \Lambda \sin \Omega t$ be the vertical location of the support. Then, from the figure, we have for the mass M

$$z_M = \Lambda \sin \Omega t + a \sin \theta \quad , \quad y_M = a \cos \theta \quad \Rightarrow$$

$$\dot{z}_M = \Omega \Lambda \cos \Omega t + a \cos \theta \, \dot{\theta} \quad , \quad \dot{y}_M = -a \sin \theta \, \dot{\theta}$$

$$T = \tfrac{1}{2} M \left(\dot{y}_M^2 + \dot{z}_M^2 \right) = \tfrac{1}{2} M \left\{ (\Omega \Lambda \cos \Omega t)^2 + 2 \Omega \Lambda \cos \Omega t \, a \cos \theta \, \dot{\theta} + a^2 \dot{\theta}^2 \right\}$$

$$V_{gravity} = Mgh = Mg z_M = Mg \sin \Omega t + Mga \sin \theta \quad , \quad V_{spring} = \text{same as before.}$$

Therefore, we now get

$$L = \tfrac{1}{2} M a^2 \dot{\theta}^2 - Mga \sin \theta - \tfrac{1}{2} k \left[2a \sin\left(\tfrac{\pi}{4} - \tfrac{\theta}{2} \right) - l_0 \right]^2$$

$$+ M \Omega \Lambda \cos \Omega t \, a \cos \theta \, \dot{\theta} \quad + \text{terms independent of } \theta + \dot{\theta}.$$