Dregt 42 cont.

$$= -(k/2)(2a^2+l_0^2) + k a l_0 \sqrt{2}\left(\frac{\sin\theta}{2} - \sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)$$

Now expand in a power series in θ. The result is

$$-(k/2)(2a^2+l_0^2) + k a l_0\sqrt{2} - k a l_0 \sqrt{2}\left(\frac{\theta}{2}\right)^2 \frac{1}{2!} + O(\theta^3)$$

Note that the linear term in θ vanishes as required by the equilibrium condition. Consequently,

$$L = \frac{1}{2} M a^2 \dot{\theta}^2 - \frac{k a l_0 \sqrt{2}}{8}\theta^2 + O(\theta^3) + \text{const}$$

$$\frac{\partial L}{\partial \theta} = -\frac{k a l_0 \sqrt{2}}{4}\theta \quad , \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = M a^2 \ddot{\theta} \Rightarrow \text{the eqn of motion}$$

$$M a^2 \ddot{\theta} + \frac{k a l_0 \sqrt{2}}{4}\theta = 0 \Rightarrow \omega^2 = \frac{k a l_0 \sqrt{2}/4}{M a^2}$$

or $$\boxed{\omega^2 = \frac{k a l_0 \sqrt{2}}{4 M a^2}}$$