So, the correct root is $Mga - ka^2 = -ka\,l_0/\sqrt{2} \Rightarrow$

$$Mga = ka^2 - ka\,l_0/\sqrt{2} \Rightarrow \boxed{M = ka/g - k\,l_0/(g\sqrt{2})}$$

Now expand L about $\theta = 0$. Need to expand

$$-Mga\sin\theta - (k/2)\left[2a\sin(\pi/4 - \theta/2) - l_0\right]^2$$ in a power series in θ.

Write $\sin(\pi/4 - \theta/2) = \sin(\pi/4)\cos\theta/2 - \cos(\pi/4)\sin\theta/2$. Then need to

expand $$-Mga\sin\theta - (k/2)\left[a\sqrt{2}\cos\theta/2 - a\sqrt{2}\sin\theta/2 - l_0\right]^2 =$$

$$-Mga\sin\theta - (k/2)\left[2a^2\cos^2\theta/2 + 2a^2\sin^2\theta/2 + l_0^2 - 4a^2\sin\theta/2\cos\theta/2 - 2a\,l_0\sqrt{2}\cos\theta/2 + 2a\,l_0\sqrt{2}\sin\theta/2\right]$$

$$= -(k/2)(2a^2 + l_0^2) + (ka^2 - Mga)\sin\theta - ka\,l_0\sqrt{2}\sin\theta/2 + ka\,l_0\sqrt{2}\cos\frac{\theta}{2}$$

$$= -(k/2)(2a^2 + l_0^2) + ka\,l_0/\sqrt{2}\,\sin\theta - ka\,l_0\sqrt{2}\sin\theta/2 + ka\,l_0\sqrt{2}\cos\frac{\theta}{2}$$