



a) $L = T - V$

$T = \frac{1}{2} M (a \dot{\theta})^2$

$V = V_{gravity} + V_{spring}$

$V_{gravity} = Mgh = Mga \sin \theta$, $V_{spring} = \frac{1}{2} k (l - l_0)^2$. From geometry,

$l/2 = a \sin (\pi/4 - \theta/2) \Rightarrow l = 2a \sin (\pi/4 - \theta/2)$. Therefore,

$V_{spring} = \frac{1}{2} k [2a \sin (\pi/4 - \theta/2) - l_0]^2$. Consequently, L is given by

$L = \frac{1}{2} M (a \dot{\theta})^2 - Mga \sin \theta - \frac{1}{2} k [2a \sin (\pi/4 - \theta/2) - l_0]^2$

The equation of motion is $\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$.

$\frac{\partial L}{\partial \dot{\theta}} = Ma^2 \dot{\theta}$, $\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = Ma^2 \ddot{\theta}$.