$$\mathcal{L} = (\dot{q}_1)^2 q_2 \cos q_3 + (\dot{q}_2)^2 q_2 q_3 + (\dot{q}_3)^2 \tan(q_2 q_3) + \dot{q}_1 \dot{q}_2 \dot{q}_3 + q_2 \exp q_3 + c t^5 q_2 q_3 .$$

a) Evidently $\partial \mathcal{L} / \partial q_1 = 0 \Rightarrow$

$$p_1 = \frac{\partial \mathcal{L}}{\partial \dot{q}_1} = 2 \dot{q}_1 q_2 \cos q_3 + \dot{q}_2 \dot{q}_3 = \text{constant}$$

b) If $c = 0$, then $\partial \mathcal{L} / \partial t = 0$ and

$$\frac{d \mathcal{H}}{dt} = \frac{\partial \mathcal{H}}{\partial t} = - \frac{\partial \mathcal{L}}{\partial t} \Rightarrow \frac{d \mathcal{H}}{dt} = 0 \Rightarrow \mathcal{H} = \text{constant}$$

$$\mathcal{H} = \sum_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \dot{q}_i - \mathcal{L} \Rightarrow$$

$$\mathcal{H} = (\dot{q}_1)^2 q_2 \cos q_3 + (\dot{q}_2)^2 q_2 q_3 + (\dot{q}_3)^2 \tan(q_2 q_3) + 2 \dot{q}_1 \dot{q}_2 \dot{q}_3 - q_2 \exp q_3 = \text{constant}$$