b) We have $p_\theta = \partial L / \partial \dot\theta = m a^2 \dot\theta \Rightarrow$

$$\boxed{\mathcal{H} = \frac{p_\theta^2}{2ma^2} - \frac{1}{2} m a^2 \omega^2 \sin^2\theta - mga\cos\theta}$$

c) $\frac{\partial \mathcal{H}}{\partial t} = 0$ providing $\frac{d\omega}{dt} = 0 \Rightarrow \frac{d\mathcal{H}}{dt} = 0 \Rightarrow$

$\mathcal{H} =$ const. As described above, $\mathcal{H} \neq E$.

d) Let $`V'(\theta) = -\frac{1}{2} m a^2 \omega^2 \sin^2\theta - mga\cos\theta$.

Expand about $\theta = 0 \Rightarrow$

$`V' = -mga + \frac{1}{2} mga\,\theta^2 - \frac{1}{2} m a^2 \omega^2 \theta^2 + O(\theta^4) \Rightarrow$

$`V' = -mga + \frac{1}{2}\left[mga - ma^2\omega^2\right]\theta^2 + O(\theta^4)$

Hamilton's eqns of motion give

$\dot\theta = \frac{\partial \mathcal{H}}{\partial p_\theta} = \frac{p_\theta}{ma^2}$, $\dot p_\theta = -\frac{\partial \mathcal{H}}{\partial \theta}$

$= -\left[mga - ma^2\omega^2\right]\theta + O(\theta^3) \Rightarrow$

$$\ddot\theta = \frac{\dot p_\theta}{ma^2} = -\frac{1}{ma^2}\left[mga - ma^2\omega^2\right]\theta + O(\theta^3)$$