Dragt 3.8 cont.

i) $\Rightarrow \quad \dot{\xi} = \dfrac{p_\rho}{\left[1 + 4(\rho_0+\xi)^2/a^2\right] m}$

$$\Rightarrow \quad p_\rho = m\left[1 + 4\rho_0^2/a^2\right]\dot{\xi} + \underbrace{\text{Higher Order Terms}}_{\text{"HOT}}$$

Put this in ii) $\Rightarrow$

$$m\left[1+4\rho_0^2/a^2\right]\ddot{\xi} = \frac{p_\phi^2}{m(\rho_0+\xi)^3} - 2mg(\rho_0+\xi)/a + HOT$$

But,

$$\frac{p_\phi^2}{m(\rho_0+\xi)^3} - 2mg(\rho_0+\xi)/a = \frac{p_\phi^2}{m\rho_0^3} - \frac{2mg\rho_0}{a}$$

$$+ \xi\left[-\frac{p_\phi^2}{m\rho_0^3}\frac{3}{\rho_0} - \frac{2mg}{a}\right] + HOT$$

Also, $\quad p_\phi^2/m\rho_0^3 = \dfrac{m^2\rho_0^4\omega^2}{m\rho_0^3} = m\rho_0\omega^2$.

$$\Rightarrow \quad \frac{p_\phi^2}{m\rho_0^3} - \frac{2mg\rho_0}{a} = m\rho_0\omega^2 - \frac{2mg\rho_0}{a}$$

$$= m\rho_0\left(\omega^2 - \frac{2g}{a}\right) = 0.$$

So, the cnst term vanishes as expected.