Dragt 38 cont.

c) $\frac{\partial \mathcal{H}}{\partial \phi} = 0 \Rightarrow p_\phi = \text{cnst}$

$\frac{\partial \mathcal{H}}{\partial t} = 0 \Rightarrow \mathcal{H} = \text{cnst}$

d) If motion is circular, then $\rho = \rho_0$, $\dot{\rho} = 0$, $\ddot{\rho} = 0$. Hamilton says

$$\dot{\rho} = \frac{\partial \mathcal{H}}{\partial p_\rho} = \frac{p_\rho}{(1+4\rho^2/a^2)m}$$

$$\therefore \dot{\rho} = 0 + \ddot{\rho} = 0 \Rightarrow \boxed{p_\rho = 0 + \dot{p}_\rho = 0.}$$

But $\dot{p}_\rho = -\frac{\partial \mathcal{H}}{\partial \rho} = \frac{p_\rho^2}{2m(1+4\rho^2/a^2)^2}\left(\frac{8\rho}{a^2}\right)$

$+ \frac{p_\phi^2}{m\rho^3} - 2mg\rho/a$. Using $\begin{matrix} p_\rho = 0 \\ \dot{p}_\rho = 0 \end{matrix}$

gives

$p_\phi^2/m\rho^3 = 2mg\rho/a$ or $p_\phi^2 = 2m^2 g\rho^4/a$

or $m^2\rho_0^4\omega^2 = 2m^2 g\rho_0^4/a \Rightarrow \boxed{\omega^2 = 2g/a}$