a) Let ρ, ϕ, z be cylindrical coordinates.

Then $T = \frac{1}{2} m \left[\dot{\rho}^2 + \rho^2 \dot{\phi}^2 + \dot{z}^2 \right]$.

Also, $x^2 + y^2 = az \Rightarrow \rho^2 = az$

$\Rightarrow z = \rho^2/a \Rightarrow \dot{z} = 2\rho\dot{\rho}/a$

$\Rightarrow \dot{z}^2 = 4\rho^2\dot{\rho}^2/a^2 \Rightarrow$

$$T = \frac{1}{2} m \left[\dot{\rho}^2 + \rho^2 \dot{\phi}^2 + 4\rho^2\dot{\rho}^2/a^2 \right] \text{ or}$$

$$T = \frac{1}{2} m \left[(1 + 4\rho^2/a^2)\dot{\rho}^2 + \rho^2\dot{\phi}^2 \right]$$

$V = mgz = mg\rho^2/a$. $L = T - V \Rightarrow$

$$\boxed{L = \frac{1}{2} m \left[(1 + 4\rho^2/a^2)\dot{\rho}^2 + \rho^2\dot{\phi}^2 \right] - mg\rho^2/a}$$