We now examine V in greater detail by constructing a contour map as shown in Figure 5. Inspection of the map shows that V vanishes on the floor of the valley (thalweg) given by equation 2.21 and at $\rho = \infty$, and it is positive elsewhere.

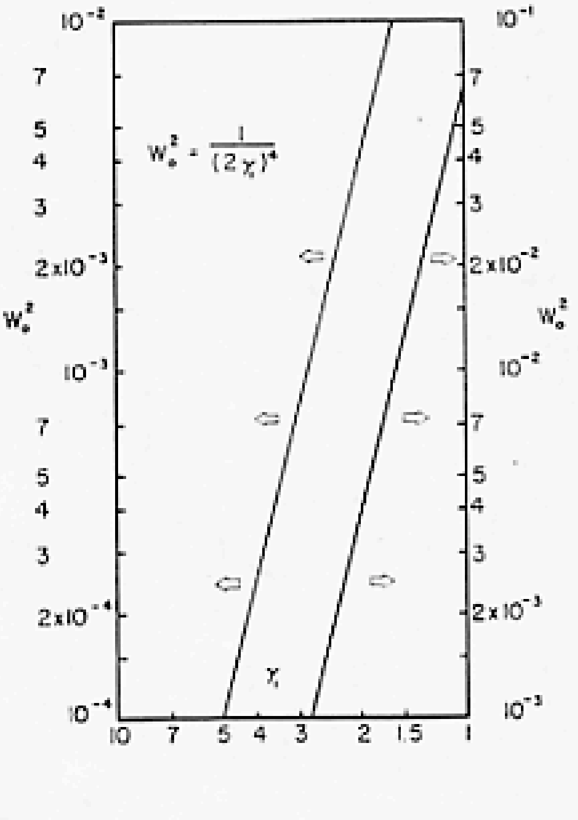

Fig. 4. The relationship between W_o^2 and γ_1.

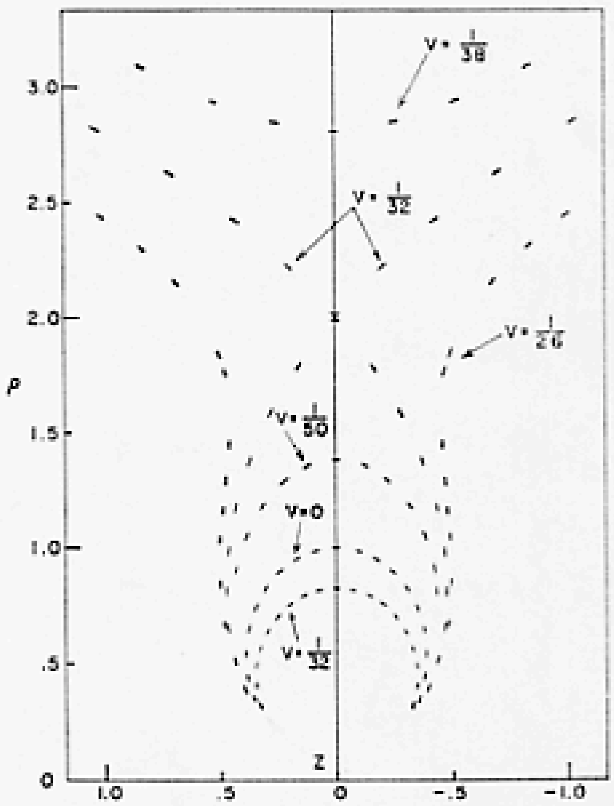

Fig. 5. A contour map of V showing lines of constant potential.

Moreover, there is a pass connecting the thalweg to infinity at $z = 0$, $\rho = 2$, where V has the value

$$V = 1/32 \tag{2.23}$$

All other paths leading from the thalweg to infinity and not going through the pass encounter larger values of V. One concludes from energy conservation that particles in the valley will be trapped provided their total energy is less than 1/32 or $\gamma_1 > 1$. These particles must remain in the vicinity of the field line given by equation 2.21 or, in the old variables, the field line given by equation 1.1. Here is the justification for our intuitive notion that low-energy particles are confined to magnetic field lines.

Unfortunately, there are no further known constants of motion, so that the system of equations 2.16–2.18 is as simple a system as one can achieve. In general, it has no known analytic solution. The equations can, however, be solved in terms of elliptic functions for the special initial conditions

$$z = \dot{z} = 0 \tag{2.24}$$

in which case the orbit is confined to the equatorial plane [*Graef and Kusaka*, 1938; *De Vogelaere*, 1950].