d) We have $p_\phi = m\rho^2\dot{\phi} + \frac{q\mu\rho^2}{[\rho^2+z^2]^{3/2}}$ 5/8

37 cont.

or $p_\phi = m\rho^2\dot{\phi} + q\rho A_\phi \Rightarrow$

$$\dot{\phi} = [p_\phi - q\rho A_\phi]/(m\rho^2) \qquad (2.4)$$

A second constant of motion follows from the invariance of $\mathcal{H}$ under rotations about the z axis

$$\dot{p}_\phi = -\partial\mathcal{H}/\partial\phi = 0 \tag{2.8}$$

or

$$p_\phi = q\mathfrak{M}\Gamma \qquad \mathfrak{M} = \mu \tag{2.9}$$

where the integration constant Γ has the dimensions of a reciprocal length.

The three-dimensional problem is thus reduced to the simpler problem of finding the two-dimensional motion of a particle in the $\rho - z$ plane under the influence of the potential

with $\gamma = 1$;

$$V = (1/2\gamma m)[(q\mathfrak{M}\Gamma/\rho) - qA_\phi]^2 \qquad \mathfrak{M} = \mu. \tag{2.10}$$

Once this problem has been solved to find $\rho(t)$ and $z(t)$, $\phi(t)$ may be found by integrating equation 2.4

with $\gamma = 1$,

$$\phi(t) = \int \frac{dt}{\gamma m\rho^2}(q\mathfrak{M}\Gamma - q\rho A_\phi) \qquad \mathfrak{M} = \mu. \tag{2.11}$$

e)

$\mathcal{L} = \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\dot{z}^2 - V(\rho,z)$ with

$$V(\rho,z) = \frac{1}{2m}\left[\frac{q\mu\Gamma}{\rho} - \frac{\mu\rho}{[\rho^2+z^2]^{3/2}}\right]^2$$