c) $\mathcal{H} = \sum_j p_j \dot{q}_j - \mathcal{L} \Rightarrow$

$$\mathcal{H} = p_\rho \dot{\rho} + p_\phi \dot{\phi} + p_z \dot{z} - \mathcal{L} \Rightarrow$$

$$\mathcal{H} = m\dot{\rho}^2 + m\rho^2\dot{\phi}^2 + \frac{g\mu\rho^2\dot{\phi}}{[\rho^2+z^2]^{3/2}} + m\dot{z}^2$$

$$-\frac{1}{2}m\dot{\rho}^2 - \frac{1}{2}m\rho^2\dot{\phi}^2 - \frac{g\mu\rho^2\dot{\phi}}{[\rho^2+z^2]^{3/2}} - \frac{1}{2}m\dot{z}^2$$

$$\Rightarrow \mathcal{H} = \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\rho^2\dot{\phi}^2 + \frac{1}{2}m\dot{z}^2 \Rightarrow$$

$$\boxed{\mathcal{H} = \frac{p_\rho^2}{2m} + \left(\frac{1}{2m}\right)\left[\frac{p_\phi}{\rho} - \frac{g\mu\rho}{[\rho^2+z^2]^{3/2}}\right]^2 + \frac{p_z^2}{2m}}$$

From the definition of A_ϕ we can also write $\mathcal{H}$ in the form

$$\boxed{\mathcal{H} = \frac{p_\rho^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2m}\left[\frac{p_\phi}{\rho} - g A_\phi\right]^2}$$

Since $\partial\mathcal{H}/\partial t = 0$, we have $d\mathcal{H}/dt = 0 \Rightarrow \boxed{\mathcal{H} = \text{cnst}}$