

## Trapped Orbits in a Magnetic Dipole Field

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*Abstract.* This article is a review of the trapped motion of a charged particle in a dipole magnetic field. It provides an extensive discussion of the equations of motion in various coordinate systems and of their solution by approximate and analytically exact methods. Wherever possible, approximate solutions are compared with exact solutions obtained by numerical integration.

### 1. INTRODUCTION

The problem of determining the motion of a charged particle in the earth's magnetic field enjoys a long history. The problem was first formulated by *Störmer* [1907] and was subsequently studied by him and several others in connection with auroral phenomena and cosmic rays. Because of this connection, particular attention was given to the orbits that enter the earth's field from infinity. More recently, owing to the discovery of the Van Allen radiation [*Van Allen et al.*, 1958], a knowledge of orbits that are trapped within the earth's field has also become important. The aim of this paper is to review the essential features of these trapped orbits in the approximation that the earth's field is replaced by that of a static magnetic dipole. (For a discussion of the more general case, see the work of *Hones* [1963], *Northrop* [1963a, b], and *Ray* [1963].)

It is convenient to begin with a qualitative description of the particle motion. Consider a low-energy particle of charge  $q$  and mass  $m$  placed in a magnetic dipole field of moment  $\mathfrak{M}$  as shown in Figure 1. Then, by intuition one expects that the

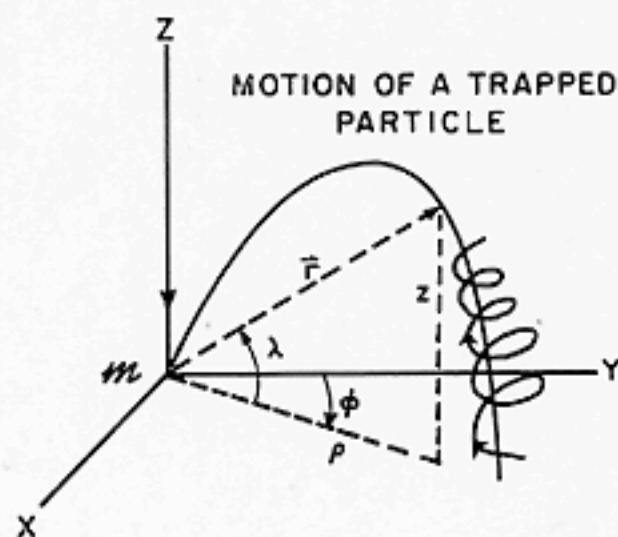


Fig. 1. Schematic representation of trapped charged particle motion in a magnetic dipole field. Note that with the conventional choice of geographic coordinates, the earth's magnetic moment  $\mathfrak{M}$  points down.

particle will gyrate about a guiding line of force

$$r = r_0 \cos^2 \lambda \quad (1.1)$$