Now A is small since $\vec{v}(0)$ is nearly $v_0 \hat{e}_z$.

Thus, to lowest order $t_\ell = \frac{\ell}{v_0}$. Putting this 3/6

in for t_ℓ we can find $x(t_\ell)$ and $y(t_\ell)$: 36 cont

$$x(t_\ell) = A \sin\delta - A \sin\left(\omega \frac{\ell}{v_0} + \delta\right)$$

$$y(t_\ell) = v_y^\circ \frac{\ell}{v_0}$$

Expand out: using $\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta$, etc.

$$x(t_\ell) = A \sin\delta - A \sin\left(\omega\frac{\ell}{v_0}\right)\cos\delta - A\cos\left(\omega\frac{\ell}{v_0}\right)\sin\delta$$

$$\boxed{\begin{aligned} x(t_\ell) &= \frac{v_0 - v_z^\circ}{\omega}\left[1 - \cos\frac{\omega \ell}{v_0}\right] + \frac{v_x^\circ}{\omega} \sin \omega\frac{\ell}{v_0} \\ y(t_\ell) &= v_y^\circ \frac{\ell}{v_0} \end{aligned}}$$

Let us write $v_z^\circ = v_0 \cos\theta$,

$v_x^\circ = v_0 \sin\theta\cos\phi \quad v_y^\circ = v_0 \sin\theta\sin\phi$

with θ small.

Then $v_z^\circ - v_0 = -v_0(1-\cos\theta) \sim \theta^2$, and

to order θ we have

$$x(t_\ell) = \frac{v_x^\circ}{\omega}\sin\omega\frac{\ell}{v_0}$$

$$y(t_\ell) = v_y^\circ \frac{\ell}{v}$$