where A_x and θ_x are arbitrary constants to be determined. By eliminating $\ddot{x}$ from Eqs. (3.301) and (3.302), in a similar way, we obtain a solution for $\dot{y}$:

$$\dot{y} = A_y \cos(\omega t + \theta_y). \tag{3.310}$$

We get x and y by integrating Eqs. (3.309) and (3.310):

$$x = C_x + \frac{at}{\omega} + \frac{A_x}{\omega} \sin(\omega t + \theta_x), \tag{3.311}$$

$$y = C_y + \frac{A_y}{\omega} \sin(\omega t + \theta_y). \tag{3.312}$$

Now a difficulty arises, for we have six constants A_x, A_y, θ_x, θ_y, C_x, and C_y to be determined, and only four initial values x_0, y_0, $\dot{x}_0$, $\dot{y}_0$ to determine them. The trouble is that we obtained the solutions (3.311) and (3.312) by differentiating the original equations, and differentiating an equation may introduce new solutions that do not satisfy the original equation. Consider, for example, the very simple equation

$$x = 3.$$

Differentiating, we get

$$\dot{x} = 0,$$

whose solution is

$$x = C.$$

Now only for one particular value of the constant C will this satisfy the original equation. Let us substitute Eqs. (3.311) and (3.312) or, equivalently, Eqs. (3.309) and (3.310) into the original Eqs. (3.301) and (3.302), using Eqs. (3.306) and (3.307):

$$-\frac{qB}{c} A_x \sin(\omega t + \theta_x) = \frac{qB}{c} A_y \cos(\omega t + \theta_y), \tag{3.313}$$

$$-\frac{qB}{c} A_y \sin(\omega t + \theta_y) = -\frac{qB}{c} A_x \cos(\omega t + \theta_x). \tag{3.314}$$

These two equations will hold only if A_x, A_y, θ_x, and θ_y are chosen so that

$$A_x = A_y, \tag{3.315}$$

$$\sin(\omega t + \theta_x) = -\cos(\omega t + \theta_y), \tag{3.316}$$

$$\cos(\omega t + \theta_x) = \sin(\omega t + \theta_y). \tag{3.317}$$

The latter two equations are satisfied if

$$\theta_y = \theta_x + \frac{\pi}{2}. \tag{3.318}$$

Let us set

$$A_x = A_y = \omega A, \tag{3.319}$$

$$\theta_x = \theta, \tag{3.320}$$

$$\theta_y = \theta + \frac{\pi}{2}. \tag{3.321}$$