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MECHANICS

THIRD EDITION

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Finally, we consider a particle of mass m, charge q, moving in a uniform constant electric field intensity $\boldsymbol{E}$ and a uniform constant magnetic induction $\boldsymbol{B}$. Again let the z-axis be chosen in the direction of $\boldsymbol{B}$, and let the y-axis be chosen so that $\boldsymbol{E}$ is parallel to the yz-plane:

$$\boldsymbol{B} = B\hat{\boldsymbol{z}}, \qquad \boldsymbol{E} = E_y\hat{\boldsymbol{y}} + E_z\hat{\boldsymbol{z}}, \tag{3.300}$$

where B, E_y, E_z are constants. The equations of motion, by Eq. (3.283), are

$$m\ddot{x} = \frac{qB}{c}\dot{y}, \tag{3.301}$$

$$m\ddot{y} = -\frac{qB}{c}\dot{x} + qE_y, \tag{3.302}$$

$$m\ddot{z} = qE_z. \tag{3.303}$$

The z-component of the motion is uniformly accelerated:

$$z = z_0 + \dot{z}_0 t + \frac{1}{2}\frac{qE_z}{m}t^2. \tag{3.304}$$

To solve the x and y equations, we differentiate Eq. (3.301) and substitute in Eq. (3.302) in order to eliminate $\ddot{y}$.

$$\frac{m^2c}{qB}\dddot{x} = -\frac{qB}{c}\dot{x} + qE_y. \tag{3.305}$$

By making the substitutions

$$\omega = \frac{qB}{mc}, \tag{3.306}$$

$$a = \frac{qE_y}{m}, \tag{3.307}$$

we can write Eq. (3.305) in the form

$$\frac{d^2\dot{x}}{dt^2} + \omega^2\dot{x} = a\omega. \tag{3.308}$$

This equation has the same form as the equation for a harmonic oscillator with angular frequency ω subject to a constant applied "force" $a\omega$, except that $\dot{x}$ appears in place of the coordinate. The corresponding oscillator problem was considered in Chapter 2, Problem 45. The solution in this case will be

$$\dot{x} = \frac{a}{\omega} + A_x\cos(\omega t + \theta_x). \tag{3.309}$$