For the Hamiltonian, suppose that:

$$\frac{\partial H'}{\partial p_i'} = \dot{q}_i \qquad \text{and} \qquad \frac{\partial H'}{\partial q_i'} = -\dot{p}_i'$$

where $p_i' = \dfrac{\partial L'}{\partial \dot{q}_i} = p_i + \dfrac{\partial \dot{F}}{\partial \dot{q}_i} = p_i + \dfrac{\partial F}{\partial q_i}$

Then
$$dH' = \frac{\partial H'}{\partial p_i} dp_i' + \frac{\partial H'}{\partial q_i} dq_i + \frac{\partial H'}{\partial t} dt$$
$$= \dot{q}_i\, dp_i' - \dot{p}_i'\, dq_i + \frac{\partial H'}{\partial t} dt$$
$$= \dot{q}_i \left(dp_i + \frac{\partial^2 F}{\partial q_k \partial q_i} dq_k + \frac{\partial^2 F}{\partial t \partial q_i} dt \right) - \dot{p}_i\, dq_i$$
$$- \frac{d}{dt}\left(\frac{\partial F}{\partial q_i}\right) dq_i + \frac{\partial H'}{\partial t} dt$$
$$= \dot{q}_i\, dp_i + \left(\cancel{\frac{\partial^2 F}{\partial q_k \partial q_i} \dot{q}_i} - \dot{p}_k - \cancel{\frac{\partial^2 F}{\partial q_k \partial q_i} \dot{q}_i} - \frac{\partial^2 F}{\partial t \partial q_k} \right) dq_k$$
$$+ \left(\frac{\partial^2 F}{\partial t \partial q_i} \dot{q}_i + \frac{\partial H'}{\partial t} \right) dt$$

Now compare this with:
$$dH' = d\left[\Sigma p_i' \dot{q}_i - L'\right] = d\left[\Sigma p_i \dot{q}_i - L + \Sigma \frac{\partial F}{\partial q_i} \dot{q}_i - \frac{dF}{dt}\right]$$
$$= dH + d\left[\cancel{\frac{\partial F}{\partial q_i} \dot{q}_i} - \cancel{\frac{\partial F}{\partial q_i}} \dot{q}_i - \frac{\partial F}{\partial t} \right]$$
$$= dH - \frac{\partial^2 F}{\partial q_i \partial t} dq_i - \frac{\partial^2 F}{\partial t^2} dt$$
$$= \frac{\partial H}{\partial p_i} dp_i + \left(\frac{\partial H}{\partial q_i} - \frac{\partial^2 F}{\partial q_i \partial t} \right) dq_i + \left(\frac{\partial H}{\partial t} - \frac{\partial^2 F}{\partial t^2} \right) dt$$